MATH-862 Finite Difference Methods for Differential Equations

Credit Hours: 3-0
Prerequisite: None

Course Objectives: The objective of this course is to find numerical solution of ordinary and partial differential equations by finite difference method. The basics and advanced topics relevant to finite difference method will be covered. These topics will be very useful for the students who opts for the research topic in differential equations. Not only students will be given theoretical aspects of numerical schemes but also programming experience in MATLAB will be helpful.

Core contents: Finite difference approximations, boundary value problems, elliptic equations, iterative method for sparse system, advection equations and hyperbolic systems

Course Contents: Truncation errors, finite difference approximations, the heat equation, the steady-state problem, local truncation error, global error, stability, consistency, steady-state heat conduction, Jacobi and Gauss-Seidal, rate of convergence, The Arnoldi process and GMRES algorithm, Advection equation, Leapfrog method, Lax-Friedrichs, The Lax-Wendroff method, Upwind methods, Von Neumann analysis, The Courant-Friedrichs-Lewy condition

Course Outcomes: After studying this subject the students will be able to:

- Compute numerical solution of ODEs and PDEs by finite difference method
- Solve sparse linear system by iterative schemes
- Program numerical solutions in MATLAB

Textbook: Finite Difference Methods for Ordinary and Partial Differential Equations by Randall J. LeVeque, Publisher: Siam, 2007.

Reference Books

1. Applied Numerical Analysis by Curtis F. Gerald and Patrick O. Wheatley, 7th Edition, Publisher: Pearson, 2003.
2. Numerical Methods for Engineers by Steven C Chapra and Raymond P Canale, 6th Edition, Publisher: McGraw-Hill, 2009.
3. Finite Difference Computing with PDEs: A Modern Software Approach by Hans Petter Langtangen and Svein Linge, Ist Edition, Publisher: Springer, 2017.

ASSESSMENT SYSTEM

Quizzes	10%
Assignments	10%
Midterms	30%
ESE	50%

Weekly Breakdown		
Week	*Sections*	*Topic*
1	1.1, 1.2, 1.3	Truncation errors, Deriving finite difference approximations, Second order derivatives
2	2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, 2.8, 2.9, 2.10	The heat equation, Boundary conditions, The steady-state problem, A simple finite difference method, Local truncation error, Global error, Stability, Consistency, Convergence, Stability in the 2- norm

3	2.15, 2.16, 2.16.1, 2.16.2, 2.16.3,	A general linear second order equation, Nonlinear equations, Discretization of the nonlinear boundary value problem, Nonuniqueness, Accuracy on nonlinear equations
4	3.1, 3.2, 3.3	Steady-state heat conduction, The 5-point stencil for the Laplacian, Ordering the unknowns and equations
5	3.4, 3.5, 3.6, 3.7, 3.7.1	Accuracy and stability, The 9-point Laplacian, Other elliptic equations, Solving the linear system, Spare storage in MATLAB
6	4.1, 4.2, 4.2.1, 4.2.2	Jacobi and Gauss-Seidal, Analysis of matrix slitting methods, Rate of convergence, Successive overrelaxation
7	4.4 4.4.1 4.4.2	The Arnoldi process and GMRES algorithm, Krylov methods based on three term recurrences, Other applications of Arnoldi
8	4.5	Newton-Krylov methods for nonlinear problems
9	**Mid Semester Exam**	
10	4.6	Multigrid methods
11	4.6.1, 4.6.2	Slow convergence of Jacobi, The multigrid approach
12	10.1 10.2 10.2.1	Advection, Method of lines discretization, Forward Euler time discretization,
13	10.2.2 10.2.3	Leapfrog, Lax-Friedrichs

14	10.3 10.3.1 10.4 10.4.1 10.4.2	The Lax-Wendroff method, Stability Analysis, Upwind methods, Stability Analysis, The Beam-Warming method
15	10.5 10.6 10.7	Von Neumann analysis, Characteristic tracing and interpolation, The Courant-Friedrichs-Lewy condition
16	10.8 10.9 10.10	Some numerical results, Modified equations, Hyperbolic systems
17	-	Review
18	**End Semester Exam**	